ACCELERATING HIGH-ORDER MESH OPTIMIZATION: METRIC LINEARIZATION AND PARTIAL ASSEMBLY

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ABSTRACT

We present a new GPU-oriented mesh optimization method for high-order finite elements [1]. The mesh optimization framework is based on the Target-Matrix Optimization Paradigm (TMOP) [2] where a mesh quality metric and geometric parameters (shape, size, and/or orientation) prescribed at each point in the domain [3] are used to define an objective function. This objective function is minimized using the Newton's method by solving a global system over the entire computational mesh. A key property of our TMOP-based approach is that the mesh optimization process is recast in terms of finite element operations, which allows us to utilize recent advances in the field of GPU-accelerated high-order finite element algorithms. Using tensor factorization and a matrix-free approach, we demonstrate improved performance compared to traditional full finite element matrix assembly. Additionally, we introduce linearization of the nonlinear TMOP mesh quality metrics, which allows us to further reduce the computational cost associated with the assembly and solve of the global system for mesh optimization. We describe the major mathematical components of the proposed method along with their efficient GPU implementation. In addition, we propose an easily reproducible mesh optimization test that can serve as a performance benchmark for the mesh optimization community.

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Keywords: Mesh optimization, High-Order, Finite elements, GPUs, Matrix-free methods

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