Meshing Ugly Geometry Using Generalized Volume Fractions

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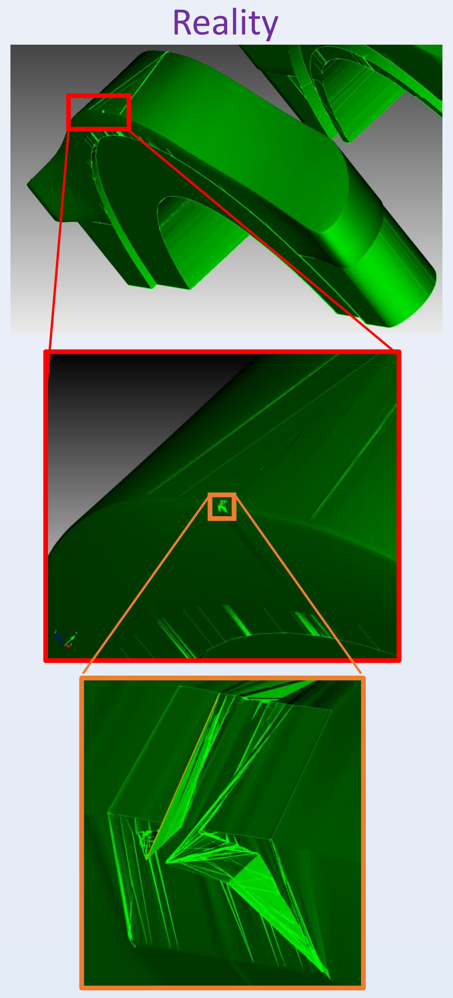
Abstract

- Two-dimensional prototype of Sculpt developed
  - Robust for ugly geometry: non-watertight, gaps, and overlaps.
- Filtration on grid size for fixed volume-fraction threshold is *non-monotonic*
  - Predicting output topology is expensive.
  - Selecting mesh size to achieve desired topology has non-continuous answers.
- Filtration on volume-fraction threshold for fixed grid size is *monotonic*
  - Sculpt topology is predictable and selectable via persistent homology.
- Shrink wrapping retains the promise of predictable and selectable mesh topology for boundary-preserving meshing algorithms.

Motivation

Fantasy vs. Reality

Geometry is Beautiful



Geometry is Ugly

Traditionally, meshing algorithms need **perfect** geometry
  Perfecting geometry takes too long and drives scientists crazy
We proposed meshing algorithms that work on **ugly** geometry
  Price is geometric and *topological* (new) fidelity to the input
  Some algorithms already do this, but with unknown topological accuracy
We proposed principled mathematics to
  Measure geometric and topological fidelity
  Guarantee fidelity bounds
    Parameterized by scientist-elected mesh size
    Successful operation despite geometric ugliness

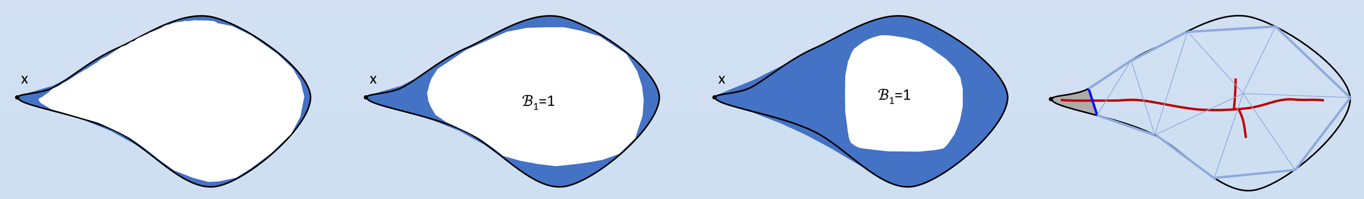
Approach

Sculpt is a volume-fraction geometric-reconstruction meshing algorithm, which in principle robustly generates meshes regardless of ugly geometry. Prior to our work, it would not run on non-watertight geometry. We sought to predict and quantify fidelity and topology using persistent homology.

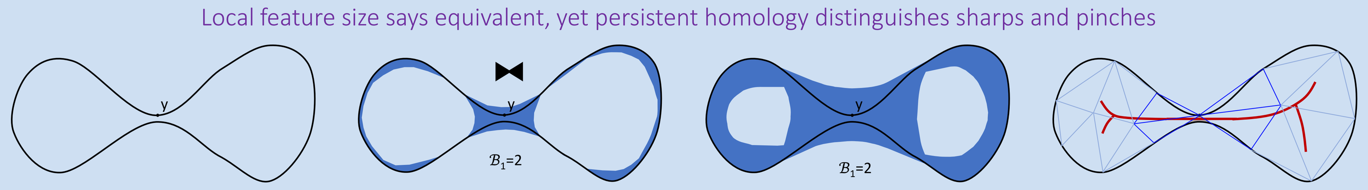
Persistent Homology (PH) measures homology changes when adding cells to a complex in a user-defined order.
Filtration: a monotonically changing cell complex based on a function-valued parameter.
Homology groups: algebraic topology groups in chain complex from boundary operator.
  $H_n = \ker(bdy_n) / \mathrm{im}(bdy_{n+1})$
  simplicial complexes and $\mathbb{Z}_1$
Betti numbers = ranks of H_n
measure different things by dimension:
B_0 = #connected components (0D)
B_1 = #internal loops (1D)
B_2= #internal cavities (2D)

<u>Characteristics of a Mesh</u>
Pick two, the third is dependent
1. Element Quality
2. Geometric & Topological Fidelity
3. Local Size (# elements)

Old School
  1 + 2 mesh matches input geometry & topology
  → 3 small elements near small features
New School
  1 + 3 mesh size is what the scientist needs
  → 2 geometry & topology is approximate



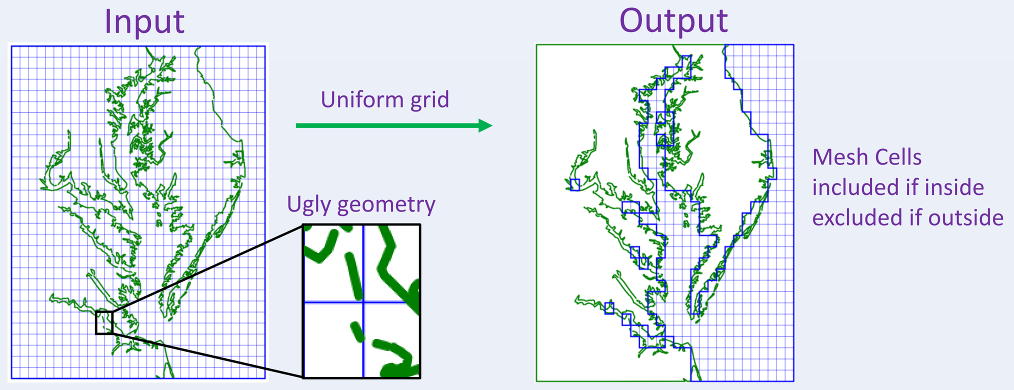
Local feature size says equivalent, yet persistent homology distinguishes sharps and pinches



Results

Thesis: Inside/outside queries on non-watertight geometry, with gaps and overlaps, can be answered by generalized winding numbers (vs. ray shooting)

Status: Implemented and demonstrated in 2D Sculpt using libigl



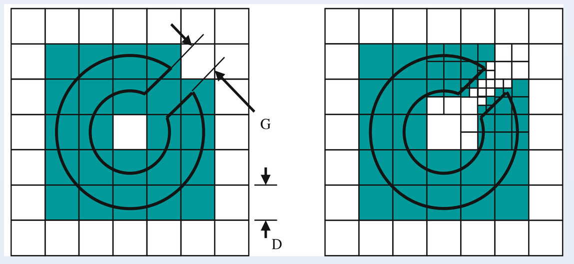
2D Sculpt meshes of Chesapeake Bay: non-watertight, gaps and overlaps

Thesis "PH-size": Persistent homology can measure the necessary mesh size to achieve a desired topology

Status: When features are isolated or globally the same scale, grid refinement has intuitive and predictable topological effects.

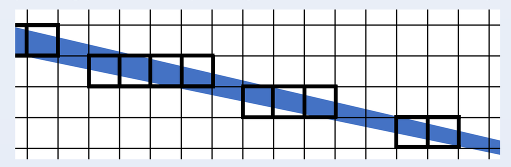
Status: Disproved for general inputs. Counterexamples show non-monotonic filtration behavior by grid size. Discretization of volume fraction by grid cells, and alignment with input features, strongly effects topological behavior.

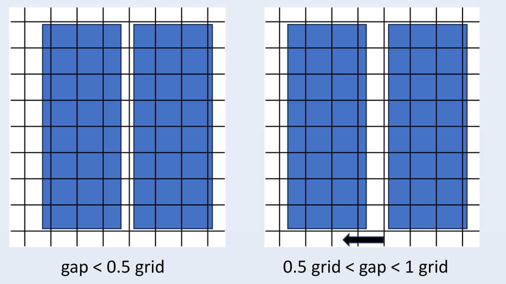
Parameters of when to refine grid have unpredictable effects on mesh topology.



Volume Fraction: cells > ½ material are filled (colored) appear in output mesh

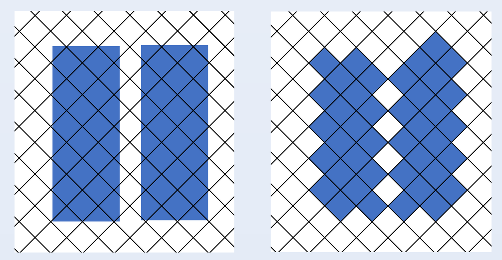
Topology of sharp angles is difficult to capture

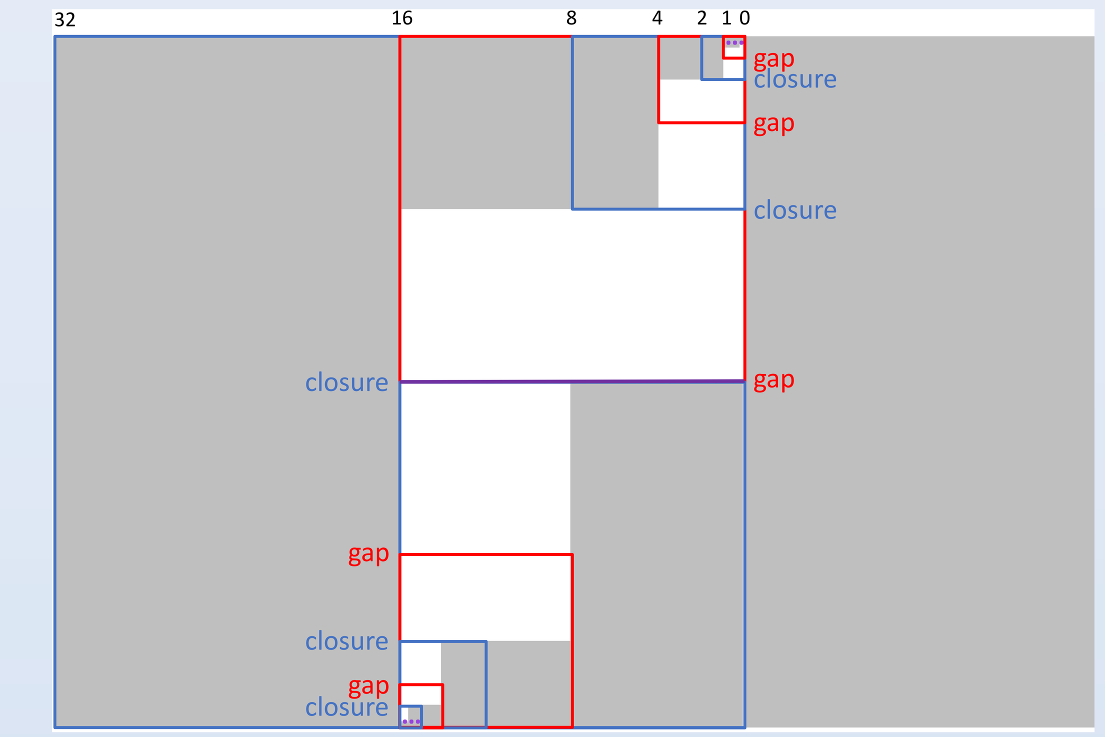




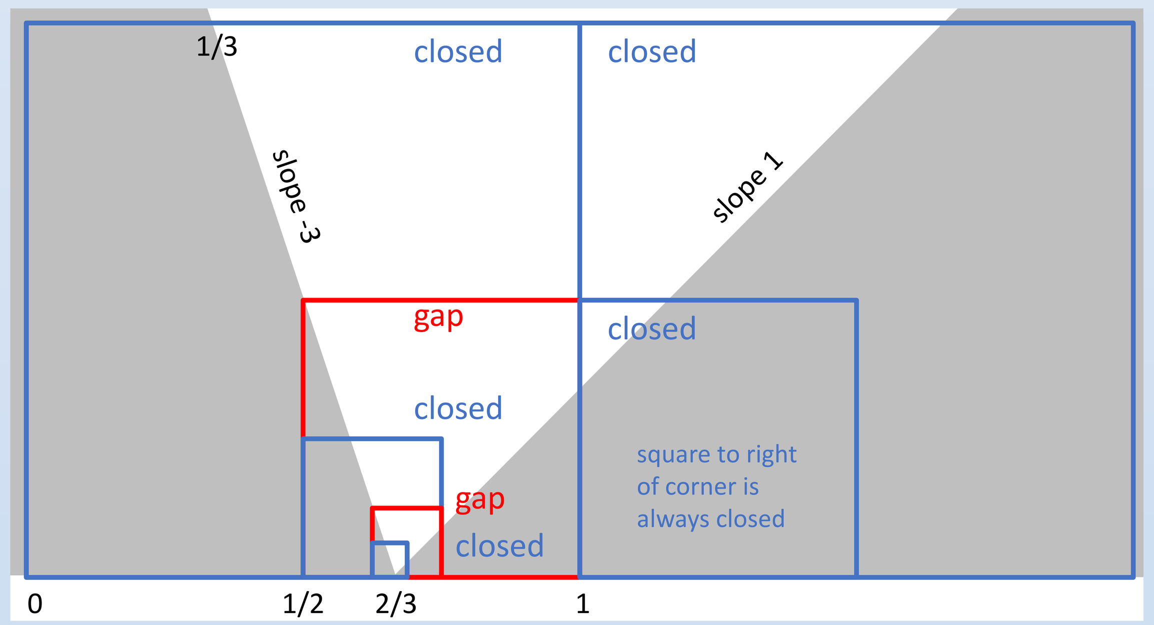
For parallel grids and gaps
- large gaps are open
- small gaps are closed
- intermediate gaps depend on alignment between grid and gap

Worse when rotating grid





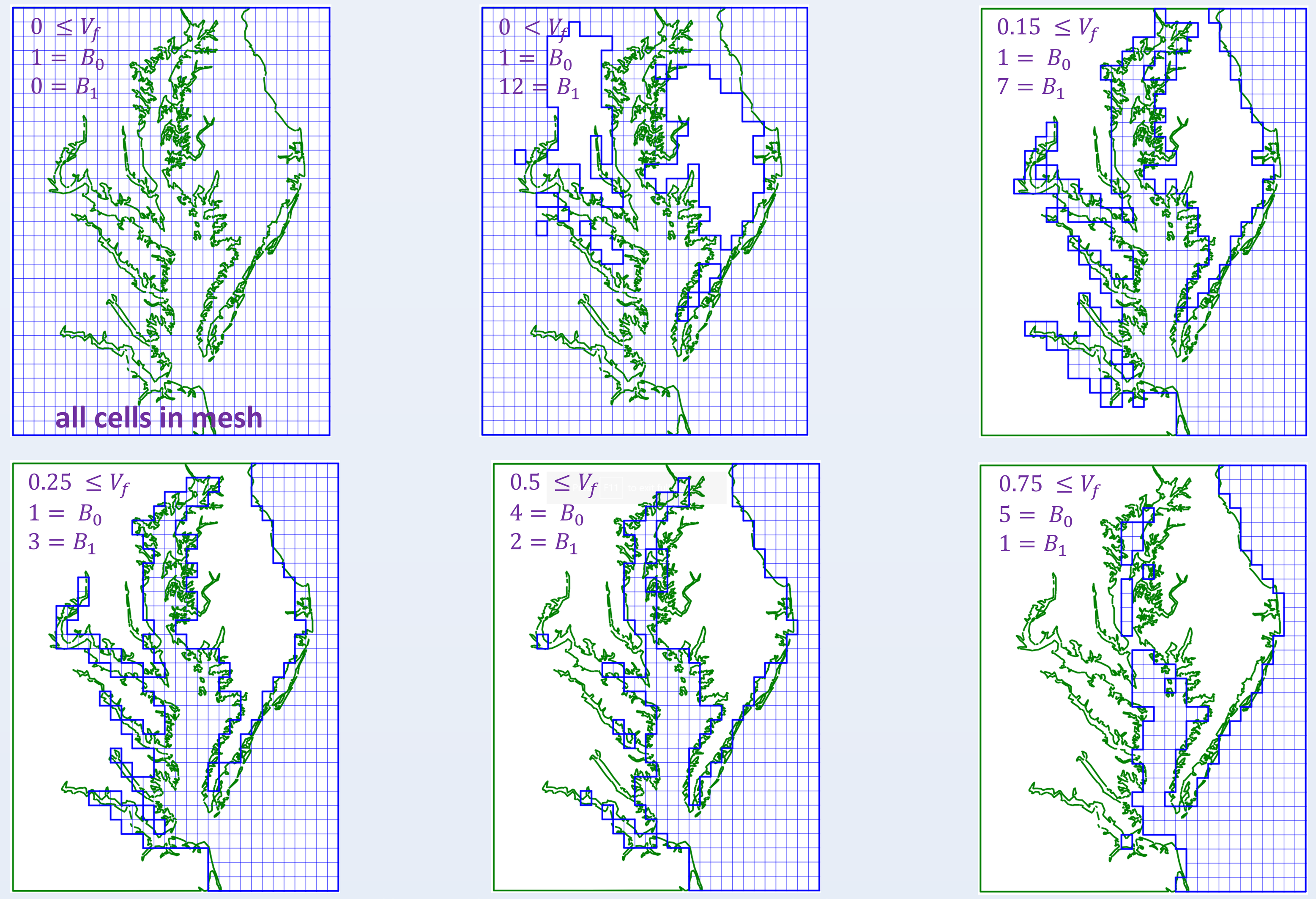
Non-monotonic: blue (closure) cells grey and filled. Red (gap) cells unfilled.
As grid size is halved, subcells alternated being filled and unfilled.



Another example of non-monotonic filtration by grid size
Volume fraction for quadrilateral mesh cells at the cusp of the grey regions oscillates between one and two connected components with mesh refinement

Thesis "PH-VC": Persistent homology measures mesh topology as volume-fraction-threshold parameter is varied: Betti barcode. Scientist picks topology, barcode shows thresholds giving it.

Status: Implemented and demonstrated in Sculpt2d. (Sculpt3d iterated cleanup heuristics for avoiding pinch points and small connected components affect the topology unpredictably.)

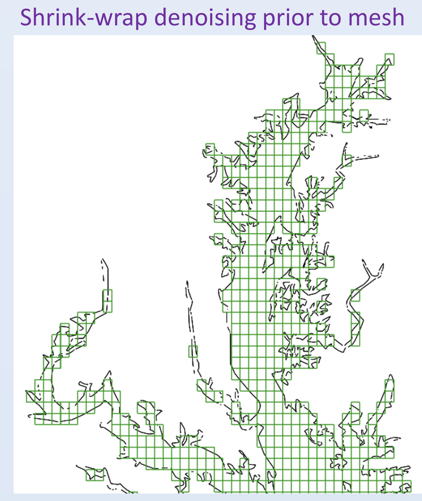


Model derived from https://vecta.io/symbols/281/ecosystems-maps/93/usa-md-va-chesapeake-bay-line-map

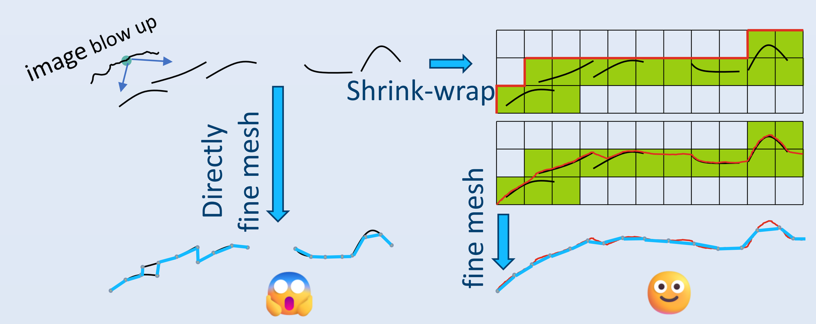
Future Work

Establish the desired homology and geometry by shrink-wrapping to maintain the topology independent of subsequent mesh size choices.

Shrink-wrap denoising prior to mesh



Enables smaller mesh size than
- Error from the image acquisition and segmentation
- Uncertainty in the exact coastline over time.



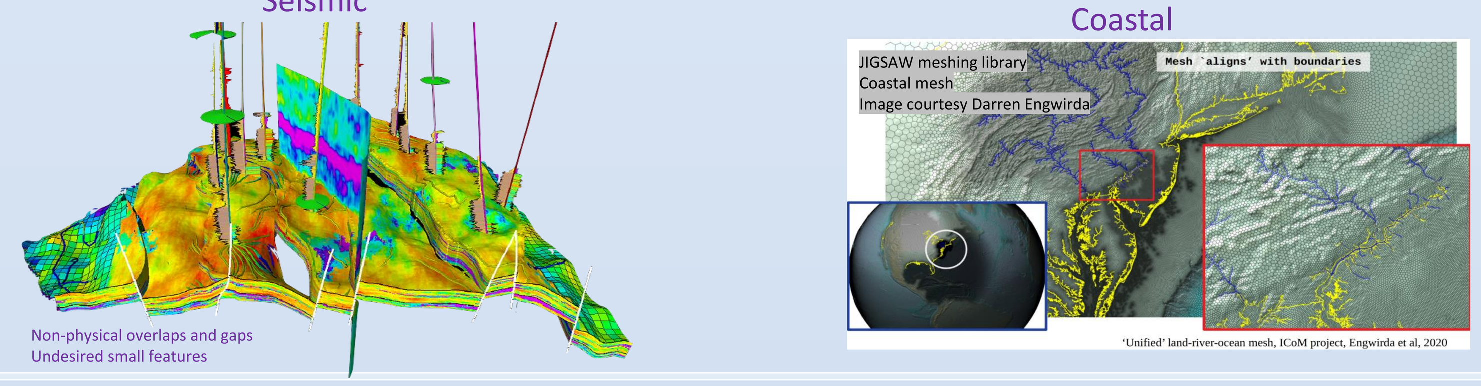
Potential Impact

Faster (human time) generation of energy and climate models with local fidelity closer to scientists' desires.

Energy and climate missions seek meshes of seismic and coastal domains.

Free scientist to
- Use any geometry available
- Select any mesh size regardless of geometric
  - Resolution
  - Errors and uncertainty

Seismic   Coastal



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- *A Shrink Wrapping Approach to Remeshing Polygonal Surfaces*, Leif P. Kobbelt et al., Comput. Graph. Forum 2001
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SAND2024-xxxxx

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